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SB-3808

M. Sc. (I.T.) (Sem. II) Examination

March / April - 2011

Paper - 201 : Mathematics - II

Time : **3** Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **M. SC. (I.T.) (SEM. 2)**

Name of the Subject :
☛ **PAPER - 201 : MATHEMATICS - 2**

☛ Subject Code No. : 3 8 0 8 ☛ Section No. (1, 2,.....) : NIL

Seat No. :

Student's Signature

(2) Attempt all questions.

(3) Use of non-programmable calculator is allowed.

(4) Figures to the right indicate full marks.

(5) Follow usual notations.

1 (a) Determine the maximum number of edges in a simple graph with n vertices and k components. **5**

OR

(a) Prove that the sum of degrees of vertices in a finite graph is twice the number of edges in it. Hence show that the number of vertices of odd degree in the graph is always even. **5**

(b) Answer any three of the following : **9**

(i) In a simple graph with n vertices, prove that the maximum degree of any vertex is (n–1)

(ii) Show that an infinite graph with a finite number of vertices must have at least one pair of vertices joined by an infinite number of parallel edges.

(iii) Define the following terms and give a suitable illustration in each case :

(a) Isomorphic graphs

(b) Ring sum of two graphs

(c) Regular graph.

(iv) Give a brief account of a history of graph theory.

(v) Find the number of vertices in a graph with 16 edges if each vertex is of degree 4.

2 (a) Let G be a complete graph with n vertices (where n is an odd number ≥ 3). Determine the number of edge disjoint Hamiltonian circuits in G. **5**

OR

(a) In a complete graph with 2k odd vertices, prove that there exists k edge disjoint sub-graphs such that they together contain all edges of G and that each is a unicursal graph. **5**

(b) Answer any three of the following : **9**

(i) If a connected graph G is an Euler graph then prove that all vertices of G are of even degree.

(ii) Describe briefly the travelling salesman problem. Using graph theory discuss the solution of it.

(iii) Define the following terms and give a suitable illustration in each case.

(a) Walk in graph

(b) Unicursal graph

(c) Fusion of two graphs

(iv) Let G be a connected graph with at least two vertices. If the number of edges in G is less than the number of vertices then prove that G has a vertex of degree one.

(v) Prove that a connected graph G remains connected after removing an edge e from G if and only if e is in some circuit in G.

3 (a) Define : **4**

(i) Vertex connectivity

(ii) Separable graph

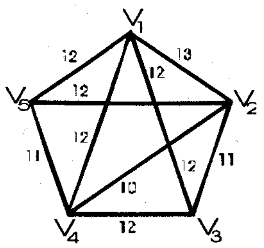
(iii) Spanning Tree

(iv) Path Matrix of a graph

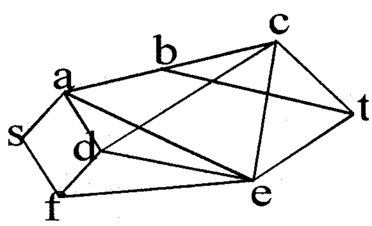
OR

(a) Define the incidence matrix of a graph with illustration and state properties of it. **4**

(b) Using the Kruskal's algorithm OR the Prim's algorithm, find a minimal spanning tree for the connected weighted graph G = (V, E) given below. **5**



(c) Apply the BFS algorithm to find the shortest path from the vertex s to the vertex t in the following graph. **5**



4 (a) Prove that every tree has either one or two centers. Determine the radius and the diameter of a regular graph with 6 vertices. **5**

OR

(a) Define a connected graph. Prove that a graph G with *n* vertices, (n–1) edges and no circuits is connected. **5**

(b) Answer any three of the following : **9**

(i) Prove that a tree with n vertices has (n–1) edges.

(ii) In a binary tree with n vertices prove that (i) max I_{max} = (n–1)/2. (ii) The number of pendant vertices is (n+1)/2.

(iii) Define the following terms with illustrations :

(a) Level of a vertex in a Binary tree

(b) Minimally connected graph

(c) Eccentricity of a vertex.

(iv) Find the rank and nullity of a complete graph with n vertices.

(v) Prove that a connected graph with n vertices and (n–1) edges is a tree.

5 (a) State and prove Euler's formula. **5**

OR

(a) Show that a complete graph with five vertices is non-planar. **5**

(b) Answer any three of the following : **9**

(i) Prove that a graph can be embedded in the surface of the sphere if and only if it can be embedded in a plane.

(ii) If the intersection of two paths in a graph is disconnected then prove that their union has at least one circuit.

(iii) Using Euler's formula, show that the Petersen's Graph is non-planar.

(iv) Let G be a connected planar graph with 6 vertices each of degree 4. Find the number of regions in G.

(v) List all possible cut-sets consisting of 3 edges in the following graph. Justify your answer.